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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL
LABORATORY, HARVARD UNIVERSITY.

AN EXPERIMENTAL DETERMINATION OF CERTAIN
COMPRESSIBILITIES.

BY P. W. BRIDGMAN.

Presented by W. C. Sabine, December 9, 1908. Received December 16, 1908.

IN a preceding paper the change of resistance produced by hydrostatic pressure on a fine thread of mercury in a capillary of a specified glass was measured. This change of resistance is the sum of two effects: the change of resistance produced by the changed dimensions of the glass capillary, and the change of resistance due to the changed electrical properties of the mercury under pressure. The change of resistance produced by the distortion of the glass is simultaneously an increase of resistance because of the decreased bore of the capillary, and a decrease because of the decreased length. The total fractional change of resistance is easily seen to be the linear compressibility of the glass. The change of resistance due to the changed electrical properties of the mercury may be further divided into two effects: that due to the change in the conducting power of the separate molecules, and that due to the change in the number of molecules occupying a given space. This latter effect is determined directly by the compressibility of the mercury.

A complete description of the phenomena involved in the measured change of resistance of the mercury involves, therefore, a knowledge of the compressibility of both the glass and the mercury. This paper is occupied with a description of the methods by which these were determined. As the pressure range employed here (6500 kgm.) is somewhat higher than that usually used, modifications of the methods in common use were necessary. It seemed undesirable, however, to bury a description of these methods in a paper on the unrelated subject of the electrical resistance of mercury, and the matter has therefore been made the subject of a separate paper, although the method has been applied to only a few substances, and all the data have been collected solely with a view to the above discussion of the effect of pressure on the resistance of mercury. However, the paper contains

an investigation of several minor points that came up in the course of the work, that may be of interest on their own account. Among these is an experimental determination of the difference of linear compressibility of a piece of commercial rolled steel along and perpendicular to the direction of rolling, and some account of the seasoning effect of successive applications of pressure on the elastical behavior of glass. In detail, the paper contains a determination by one method of the compressibility of two kinds of Jena glass, of a piece of commercial aluminum rod, and of several grades of steel; and by another method, the compressibility of mercury, all up to about 6500 kgm. per sq. cm.

In determining the compressibility of a solid the method adopted was to measure the change of length of a rod of the substance produced by hydrostatic pressure applied all over the external surface. This method applies, therefore, only to those solids that can be obtained in the form of a cylindrical rod or tube. The cubic compressibility is found by multiplying the linear compressibility by three. It is a fundamental assumption throughout all the following determinations of the compressibility of solids, therefore, that the substance is so homogeneous and isotropic that the compression under hydrostatic pressure is sensibly the same in all directions. Some experimental proof of the justifiability of this assumption has been attempted in the case of a piece of rolled steel boiler plate.

It is a feature common to all the compressibility methods used in this paper, that the distortion produced by pressure is measured by the displacement of a ring sliding with slight friction on some movable part of the apparatus. The method is not continuous reading, therefore, but the apparatus has to be taken apart and readings made after each application of pressure, the reading obtained corresponding to the maximum pressure. A method of this kind is doubtless inconvenient, but it has the advantage of simplicity and directness over any continuous reading method that would be practical over so wide a pressure range.

In the determination of the compressibility of solids two slightly different methods may be used, according as the solid is of relatively low or high compressibility. The first method, not so accurate as the second, applies to iron and metals of the same order of compressibility. The second applies to substances of higher compressibility, and involves directly the compressibility of iron as determined by the first method.

The first method measures the relative change of length of a rod of the substance and a heavy cylinder of steel. The rod is enclosed in the cylinder, throughout the interior of which hydrostatic pressure

is applied. The rod shortens, therefore, under the uniform external pressure, while the cylinder lengthens under the interior pressure. The lengthening of the cylinder is very much less than the shortening of the rod. In the present experiment it was only 5 per cent. The strain in the cylinder is complicated, consisting of a radial displacement away from the centre, and of a longitudinal extension which may produce warping of the originally plain sections. This warping is greatest at the ends, and must vanish at the mid section if the cylinder is symmetrical at the two ends. The warping cannot be easily calculated, and was neglected in the present work. It can in any event constitute only a correction for the above 5 per cent correction term. The method consists, therefore, in subtracting from the

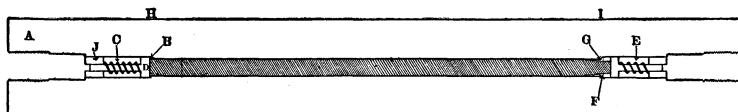


FIGURE 1. Apparatus for measuring the linear compressibility of rods. The rod to be measured is indicated by the shading. The stop D is held permanently against the shoulder B by the spring C, which is kept compressed by the pump connections, not shown. The brass ring F is kept in contact with the shoulder G during increase of pressure by the spring E, which pushes the shortening rod through the ring F, so as always to be in contact with the stop D. When pressure is released the ring comes back with the rod and the displacement is measured. The rod is removed through the end E to make these measurements; the connections at A to the pressure pump are not disturbed during the measurements. The elongation of the cylinder is measured externally at the scratches H and I.

relative change of length of the rod and the cylinder the increase of length of the cylinder as obtained from the measurement of external change of length under pressure. The result is the linear compressibility of the rod, from which the cubic compressibility is calculated.

The cylinder used is shown in Figure 1. It is made of annealed tool steel, 18 in. (45.7 cm.) long, and 2 in. (5.1 cm.) in diameter. It is pierced through the entire length by a reamed $\frac{3}{8}$ in. (0.95 cm.) hole, in which the rod to be tested is placed. At either end the $\frac{3}{8}$ in. hole is enlarged in several steps in the manner indicated, in order to afford room for the various connections. The enlargements of the holes are precisely alike at the two ends, so as to insure symmetrical warping of the cylinder. The rod to be tested is indicated by the shading. It is carefully turned so as to slide without lateral play into the reamed hole. Three shallow grooves, milled the entire length of this rod,

allow the compressing fluid to flow freely from one end of the cylinder to the other. The change of length of the rod is obtained by keeping one end of the rod always fixed opposite the same part of the cylinder, and measuring the relative displacement of the other end, which is free. The fixed end of the rod is kept so by the action of the spring at E, which keeps the rod pressed against the stop at D. This stop D is kept immovable by the spring at C, which keeps D pressed against the shoulder B. This spring C is very much stiffer than the spring E, and is kept permanently compressed by the pump connections (not

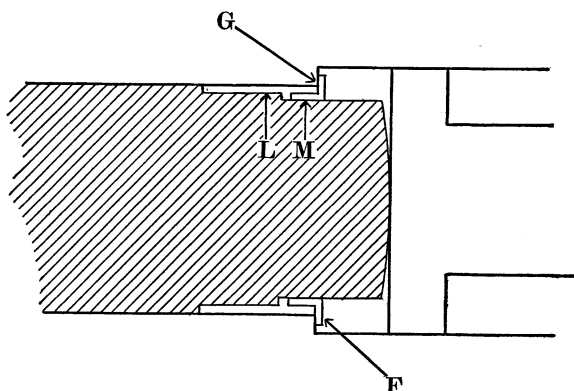


FIGURE 2. Enlarged view of the brass ring, etc., of Figure 1. The displacement of the ring is measured by measuring the distance between the scratches at L and M on the rod and the ring respectively.

shown) which are screwed into the end A, and keep the ring J fast in the position shown. This method of securing the invariable position of the stop seemed preferable to any plug arrangement screwed fast into the cylinder, for the latter might shift slightly, owing to the change produced by the pressure in the dimensions of the thread.

The shift of the free end of the rod relatively to the cylinder was obtained by measuring the displacement on the brass ring F, which is pushed back by the shoulder G. An enlarged view of the ring is shown in Figure 2. The brass ring F is split so as to slide without too great friction on the end of the rod, which is turned down to about $\frac{1}{8}$ in. (8 mm.). There is a fine scratch on the ring at M, and also a scratch on the corresponding ledge L of the rod. The ring and rod are turned in the lathe so that these two scratches are at the same radial distance from the axis of the rod, thus enabling both scratches to be in focus simultaneously under a high power microscope. The effect of an application of pressure is to shorten the rod, pushing

up the ring, which stays in its extreme position. The rod is then taken out by unscrewing the plug at the end I and the distance between the scratches L and M measured. The increase of distance over the zero position gives the relative change of length of rod and cylinder. There is here a small source of error in finding the effective length of the rod, which terminates at some unknown place within the brass ring. The effective length used was the length from the fixed end to the middle of the ring when in the zero position. As the breadth of the bearing surface of the ring was only about 2 mm., and the length of the rod was 30 cm., the maximum error here is only $1/300$.

It is at once obvious that any slight error in replacing the rod after each measurement in exactly its former position will produce considerable error in the result, since the change of length produced by pressure is small. In the form used, in which the rod is 30 cm. long, the change of length for 1000 kgm. is only 0.05 mm. Slight particles of grit are likely, therefore, to produce considerable irregularities. By working with some care it was found possible, however, to secure fairly uniform results. Particular attention must be given to washing out the cylinder after each application of pressure. The effect of pressure is, of course, to flood the interior of the cylinder with the pump liquid, in this case glycerine and water, which may carry considerable grit in suspension. After each measurement the cylinder was thoroughly washed several times by a jet of water violently expelled from a glass tube reaching into the cylinder as far as the stop D. No cloth or other substance must be used for wiping out the hole. The rod to be tested was also carefully washed under the tap after each measurement, again taking care not to wipe with a cloth or to bring into contact with any possible source of grit. It was found that by decreasing the diameter of the rod for a short distance at the end B, there is less tendency for grit to collect between the end of the rod and the stop D when the rod is replaced in the hole after each measurement.

The change of length of the steel cylinder was not measured at the same time as the relative change of length of rod and cylinder, but was, instead, determined independently as a function of the pressure. Three determinations of this extension were made, one preliminary to, one in the course of, and one after the series of compressibility measurements. The last two agreed within the limits of error; the first was slightly different, as has always been found to be the case when the deformation of a metal is measured on the first application of pressure. In making these measurements, the cylinder was clamped to a heavy comparator bed, which carried two microscopes. The cylinder was clamped at only one point, the middle, so as to avoid

any possible distortion of the comparator by the lengthening of the cylinder under pressure. The close contact of cylinder and comparator insured the practical equality of temperature of the two, and the coefficients of expansion of the two pieces proved so close that the few tenths of a degree variation which occurred in the temperature of the room introduced no appreciable error. The microscopes were focussed on fine fortuitous scratches on the cylinder at the points H and I (Figure 1). Change of length was measured by a micrometer eyepiece in either microscope, which had been previously calibrated. Settings on the fine scratches could be made with a maximum error of 0.0003 mm.,

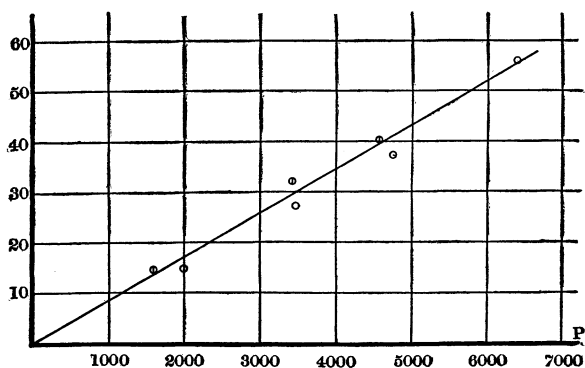


FIGURE 3. The elongation of the cylinder of figure 1, as a function of the pressure. $\bigcirc$, observations at increasing pressures; $\odot$, at decreasing pressures. The ordinates give the proportional elongation multiplied by 10^6 . That is at a pressure of 6400 kgm. per sq. cm. the elongation of the cylinder is 0.000056 per unit length.

thus introducing a possible error of reading of the change of length of 0.0006 mm. The total change of length was found to be 0.02 mm. at 6000 kgm. The maximum error here possible on the extension coefficient of the cylinder is, therefore, 6 parts in 200. The mean of several readings, of course, has a much less probable error.

The results obtained are shown in Figure 3, in which extension of the cylinder is plotted against pressure. The pressure was measured here, as in all subsequent work in this paper, by a secondary gauge depending on the variation of the resistance of mercury under pressure. The justification and calibration of this gauge has been made the subject of another paper. The figure shows distinct evidence of hysteresis, the extension under decreasing pressure being greater than the corresponding extension under increasing pressure. This is the more surprising as the total extension of the cylinder is only $\frac{1}{30}$ of the value of the

extension at the elastic limit under pure tension. The departure of the points from a straight line representing the mean is comparatively slight, however, and in applying the corrections determined in this way the relation between extension and pressure was assumed to be linear.

With this apparatus the linear compressibility of a piece of commercial aluminum rod and several specimens of iron and steel were made. In Figure 4 is shown the fractional change of length of the aluminum rod corrected for the extension of the steel cylinder, plotted against pressure. This figure does not include the first observation which was made with a pressure slightly higher than any subsequently reached. The rod took a distinct set on this first application, being permanently shortened by one part in 30,000. No evidence of further set was found on subsequent applications of pressure. This is the first occasion on which a set in any dimension by the application of hydrostatic pressure has been directly observed. No attempt was made to find whether this linear set is accompanied by volume set. The displacement was measured from the mean of several determinations of the position of the ring at zero pressure. But this determination is obviously affected by the same errors as displacement measurements at higher pressures. It is evident from the figure that within the limits of error the points lie on a straight line. This was assumed to be of the form $a + bp$, and a and b determined by least squares, discarding the most discordant results. a is the true zero position and b the pressure coefficient of contraction. In this way every measurement at any pressure contributes to the more accurate determining of

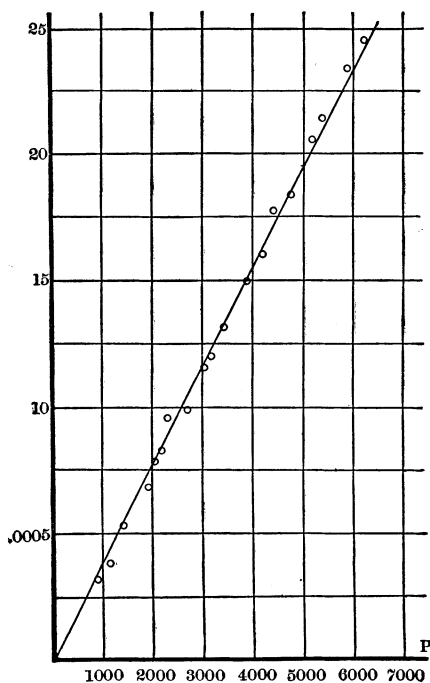


FIGURE 4. The observed proportional change of length of an aluminum rod plotted against pressure.

TABLE I.
COMPRESSIBILITY OF ALUMINUM ROD.

Order of Observation.	Pressure kgm./cm. ²	$\frac{\Delta l}{l_0}$.		
		Observed.	Calculated.	Difference.
18	900	0.000320	0.000346	+26
6	1154	0.000386*	0.000445	+59
12	1436	0.000545	0.000555	+10
5	1910	0.000684*	0.000741	+57
19	2050	0.000790	0.000796	+ 6
11	2180	0.000867	0.000847	-20
17	2396	0.000960	0.000910	-50
4	2694	0.000990*	0.001048	+58
13	3030	0.001163	0.001179	+16
7	3180	0.001202	0.001238	+36
8	3416	0.001318	0.001330	+12
1	3890	0.001509	0.001515	+ 6
10	4230	0.001605	0.001648	+43
14	4418	0.001775*	0.001722	-53
2	4760	0.001838	0.001855	+17
9	5200	0.002054	0.002027	-27
16	5384	0.002140	0.002099	-41
3	5892	0.002339	0.002297	-42
15	6240	0.002450	0.002434	-16
$\frac{\Delta l}{l_0} = a + bp.$ $a = -0.0000056.$ $b = 0.0000003910.$ * Discarded in the calculation.				

the zero position, the necessity of a large number of determinations of which are therefore avoided. It was found that

$$\frac{\Delta l}{l_0} = -0.0000056 + 0.0000003910 p.$$

The cubic compressibility is, therefore, 0.000001173 kgm. per sq. cm. In Table I are shown the observed and calculated results. The probable error of a single observation is less than one per cent at the higher pressures. The probable error of b , the compressibility, is about $\frac{1}{3}$ per cent. The value found by Richards¹ for the compressibility of aluminum is 1.28×10^{-6} . He does not state the chemical purity of the aluminum. The specimen used above was commercial aluminum rod, which is usually very pure. No chemical analysis was made, however, and the discrepancy may be due to impurities.

In an exactly similar manner the compressibilities of several samples of iron or steel were determined. The first piece was from a piece of $\frac{1}{2}$ in. (1.27 cm.) Bessemer rod annealed by heating to redness and cooling slowly, and then turned down to $\frac{3}{8}$ in. (0.95 cm.). It was from the same piece of rod as a piezometer for determining the compressibility of mercury, as will be described later. The results obtained for this steel corrected for the extension of the cylinder are plotted in Figure 5, the zero being arbitrary as formerly. The results are better proportionately than for

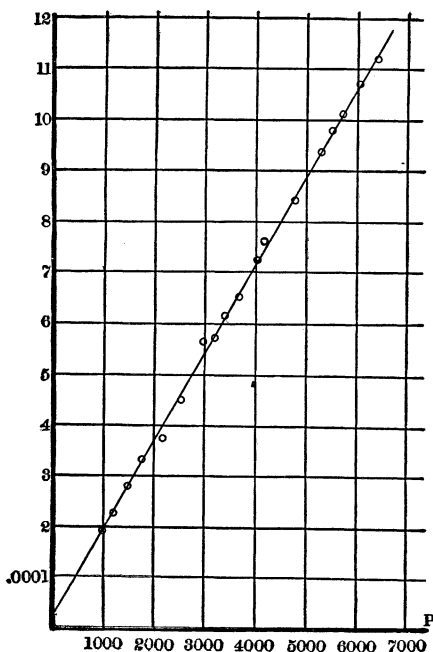


FIGURE 5. The observed proportional change of length of a rod of Bessemer steel plotted against pressure. The zero is here arbitrary.

¹ Compressibilities of the Elements and their Periodic Relations. Richards, Carnegie Inst., Washington, p. 61 (1907).

TABLE II.
COMPRESSIBILITY OF BESSEMER ROD. SAME MATERIAL
AS MERCURY PIEZOMETER.

Pressure kgm./cm.	$\frac{\Delta l}{l_0}$		
	Observed.	Calculated.	Difference.
994	0.000195	0.000196	+ 1
1190	0.000228	0.000230	+ 2
1488	0.000281	0.000281	..
1770	0.000332	0.000330	- 2
2174	0.000374*	0.000399	+25
2540	0.000452*	0.000462	+10
2980	0.000565*	0.000538	-27
3176	0.000570	0.000572	+ 2
3400	0.000615	0.000611	- 4
3670	0.000652	0.000657	+ 5
4040	0.000724	0.000721	- 3
4176	0.000769*	0.000744	-25
4760	0.000839	0.000845	+ 6
5294	0.000938	0.000937	- 1
5506	0.000977	0.000973	- 4
5730	0.001013	0.001012	- 1
6060	0.001072	0.001068	- 4
6430	0.001127	0.001133	+ 6
$\frac{\Delta l}{l_0} = a + bp.$ $a = 0.0000249.$ $b = 0.0000001722.$ * Discarded in the calculation.			

the aluminum, although, because of the smaller size of the total effect, one would expect greater percentage variation from the particles of grit. Probably the improvement is due to the increased familiarity with the method, which seems capable of giving accurate results. A straight line through the observations, discarding the four worst, was computed by least squares, giving as the linear compressibility 1.722×10^{-7} , and the cubic compressibility 5.166×10^{-7} kgm. per sq. cm. Table II shows the differences between the observed and the computed values. The four starred points are the ones discarded in the computation. The probable error of a single observation, excepting the four discarded ones, is 2.3, less than $\frac{1}{4}$ per cent at the higher pressures. The probable error of the compressibility is $\frac{1}{10}$ per cent, which therefore does not vary more than this from constancy throughout the pressure range. No set was observed in this piece of steel on the first application of pressure, which is perhaps evidence of the freedom from internal strain, and to a less degree evidence for equal compressibility in all directions.

An attempt was made to get some light on the possible magnitude of differences of compressibility in different directions by the following method: Two strips were cut from a very homogeneous piece of $\frac{5}{8}$ in. (1.59 cm.) Bessemer boiler plate, respectively along and perpendicular to the direction of rolling; these were turned down to $\frac{3}{8}$ in. (0.95 cm.) like the other test pieces of steel or aluminum, and the compressibility of each determined. The results are given in Tables III and IV. The compressibility of each was calculated by least squares, discarding only one observation from each set. The probable error of a single observation is approximately the same in either set, $\frac{1}{10}$ per cent at the higher pressures. The probable error of the compressibility in either case is about $\frac{1}{10}$ per cent. The compressibility of the lengthwise piece was 5.298×10^{-7} , and of the transverse 5.303×10^{-7} , agreeing within the limits of error. No claim is made that this settles the question of the equal compressibility of metals in all directions. Doubtless with metals of different character there are internal strains left from working that would produce such a difference.

There are only a few other direct determinations of the compressibility of steel. Amagat² measured the change of length by an electric contact device, but does not publish his data. He states that the results agree with a determination by an indirect method involving the theory of elasticity and gives 6.8×10^{-7} as the best value. Richards,³

² Amagat, C. R., 108, 1199 (1888).

³ Richards, loc. cit., p. 50.

COMPRESSIBILITY OF BESSEMER BOILER PLATE.

TABLE III. LONGITUDINAL.

TABLE IV. TRANSVERSE.

Pres- sure kgm. cm. ²	$\frac{\Delta l}{l_0}$			Pres- sure kgm. cm. ²	$\frac{\Delta l}{l_0}$		
	Obs.	Calc.	Diff.		Obs.	Calc.	Diff.
794	0.000120	0 000120	0	1000	0.000174	0.000173	- 1
984	0.000156	0.000154	- 2	1190	0.000197	0.000206	+ 9
1150	0.000176	0.000186	+10	1222	0.000215	0.000212	- 3
1396	0.000233	0.000227	- 6	1446	[0.000192]	0.000252	+60
1660	0.000271	0.000273	+ 2	1680	0.000280	0.000293	+13
2016	0.000314	0.000336	+22	2014	0.000345	0.000353	+ 8
2228	0.000362	0.000373	+11	2180	0.000384	0.000381	- 3
2480	0.000431	0.000418	-13	2526	0.000439	0.000442	+ 3
2834	0.000481	0.000480	- 1	2816	0.000518	0.000494	-24
3040	0.000500	0.000517	+17	3060	0.000537	0.000537	0
3272	0.000591	0.000558	-33	3346	0.000593	0.000586	- 7
3540	[0.000663]	0.000595	-68	3660	0.000635	0.000643	+ 8
3646	0.000625	0.000624	- 1	3980	0.000703	0.000699	- 4
3920	0.000672	0.000672	0	4186	0.000729	0.000736	+ 7
4398	0.000748	0.000757	+49	4472	0.000789	0.000786	- 3
4400	0.000761	0.000757	- 4	4988	0.000906	0.000877	-29
4740	0.000835	0.000817	-18	5294	0.000929	0.000932	+ 3
4920	0.000847	0.000849	+ 2	5456	0.000966	0.000960	- 6
5340	0.000954	0.000923	-31	5668	0.000994	0.000998	+ 4
5440	0.000938	0.000941	+ 3	6034	0.001044	0.001063	+19
5690	0.000988	0.000985	- 3	6210	0.001099	0.001093	- 6
6164	0.001061	0.001069	+ 8	6400	0.001099	0.001128	+29
6430	0.001099	0.001116	+17	...			..
$\frac{\Delta l}{l_0} = a + bp.$ $a = -0.000020.$ $b = \log^{-1} 3.2470 - 10.$ Cubic compressibility = 0.065298.				$\frac{\Delta l}{l_0} = a + bp.$ $a = -0.000004.$ $b = \log^{-1} 3.2474 - 10.$ Cubic compressibility = 0.065303.			

also observing the change of length by an electrical contact device, finds 3.9×10^{-7} . The iron used by Richards was commercial wrought iron, chemical analysis of which is not given. The mild Bessemer steel used in this investigation is usually as free from carbon as wrought iron, and is very much more likely to be homogeneous. The absence of set is evidence of the closeness of texture, while Richards states that the wrought iron used by him was porous and had to be hammered to give satisfactory results. This possibly may account for some of the difference in the results.

To get some idea of the effect of varying percentage of carbon, the compressibility of a piece of high carbon (1.25 per cent) annealed tool steel was determined with the same probable error as in the other determinations, and was found to be 0.000000525. The discrepancies between Richards' values and the values found in this paper can hardly be explained by impurities of this nature.

It is to be noted that neither the steel nor the aluminum shows any tendency to become decreasingly compressible at higher pressures, in analogy with the behavior of more compressible substances, particularly liquids. In fact, as will be seen from an inspection of either the curves or the table, the aluminum shows a distinct though slight tendency to become more compressible at higher pressures. However, it did not seem that this single example would justify the conclusion that this paradoxical behavior was due to anything except errors of observation, and accordingly the coefficient was calculated by least squares on the assumption that it was constant.

The second modification of the above method for measuring linear compressibility consists in comparing the change of length of a tube of the substance in question with the simultaneous change of length of a piece of steel, both the substance and the steel suffering uniform contraction by the hydrostatic pressure over the whole exterior surface. From the relative change of length the absolute linear compressibility may be found if the linear compressibility of the comparison piece of steel is known. This latter may be found by the first method given above.

The apparatus with which the relative change of length of the tube (in this case of glass) and the steel were determined is shown in Figure 6. The glass tube C was kept pressed against the bottom B of the cylindrical hole in the steel cylinder A, by the spring at G, through the medium of the tie rod H and the nuts E and F. A split brass ring D slides on the glass tube easily, but tightly enough to remain securely in position under moderate jarring. Fine scratches were made on the steel at I and the flange of the brass ring. The whole combination was

placed in the pressure chamber, and subjected to hydrostatic pressure all over. Both glass and steel shrink, the glass shrinking the more, and hence the ring D is pushed up on the tube. When pressure is released, D comes back with the tube, and the increased distance between the scratches, measured with two microscopes, gives the relative change of length for the highest pressure reached. The glass tube was taken out of the steel jacket and everything washed carefully after each application of pressure, in order to insure freedom from small particles of grit. It is an advantage of this method over the first, that because of the greater accessibility of the parts, complete freedom from grit is secured by washing after each application of pressure. Repeated measurements of the zero position of the ring gave results agreeing within 0.001 mm., which in this case was about the magnitude of possible errors of reading. The total displacement at 6500 kgm. was about 0.35 mm. in the form above.

Among possible sources of error we have here again a maximum uncertainty in the effective length of the glass tube of $\frac{1}{2}$ the width of the ring D. In the form used the total length was about 8 cm., and the width of the ring 2 mm. The results may, therefore, be in error by $\frac{1}{80}$, but probably by less than this. This source of error may obviously be decreased at pleasure by increasing the length of the tube. Another possible source of error is temperature change. Error from heat of compression was avoided by operating slowly, applying pressure nearly to the maximum, waiting

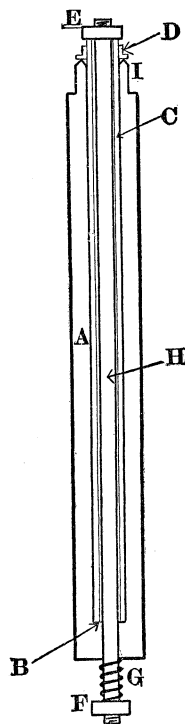


FIGURE 6. Apparatus for comparing the linear compressibility of glass and steel. The glass tube C is compared with the enveloping steel tube A. The relative change of length is measured by measuring the displacement of the ring at D, sliding on the glass tube. The glass tube is kept in contact with the shoulder B by the spring G, acting on the nut F through the tie rod H, which in turn presses on the glass tube by the nut E.

for the equalization of the temperature, and then applying the last few hundred kilograms. Differences of temperature at different times of measuring the displacement did not prove great enough to introduce perceptible error, since the difference of dilation between the glass and the steel is small. To secure good results, it was found necessary that the glass tube fit closely in the steel cylinder without play sidewise. As it was found difficult to draw a tube accurately enough, this desired freedom from play was secured by wrapping tin foil at either end.

Measurements were made in this way of the change of length of two kinds of Jena glass: a hard combustion tubing No. 3883, and a very fusible glass No. 3880 a. The results at first were discouragingly irregular. After repeated trials, however, they settled down into a fairly regular final form. It became evident on trial with different pieces of glass that there is here, directly observed, the same seasoning effect of successive applications of pressure that was noticed in measurements of electrical resistance. The final behavior never became entirely regular, however. The general effect of frequent applications was to slightly increase, in a totally irregular fashion, however, the observed change in length. In Figure 7 the observed changes of length are plotted against pressures. The irregularity of the results is noticeable, particularly for the hard glass; it approaches, or may sometimes exceed, 5 per cent of the total effect to be expected. The results with the soft Jena glass were only one third as irregular. The explanation suggests itself that the less regularity of the results with the hard infusible glass is because of the greater internal strains set up in this by the long temperature range through which it cools after passing plasticity.

In order to find whether there is any appreciable change in the linear compressibility of glass when it is drawn down from larger sizes, the above form of apparatus was modified by placing the comparison piece of steel inside, instead of outside, the glass tube. The tubes tested were 1 cm. in diameter, which is the original size from which the test pieces mentioned above were drawn down to 0.5 cm. Within the limits of error, no variation of compressibility with absolute size could be detected.

The linear compressibility of the steel against which the glass was compared was determined indirectly by finding the relative change of length in the same manner as for the glass, of the steel and a piece of aluminum cut from the rod whose absolute compressibility was determined above. These readings of the relative change of steel and aluminum are shown in the lower line in Figure 7. The points, except

one, lie on a straight line within errors of reading. The one discordant point represents a discrepancy of only 0.0003 mm., and no importance is attached to it. The regularity of these measurements of the aluminum, made with the same apparatus as the measurements of the glass, furnishes additional presumptive evidence, therefore, that the irregularity of the latter is not due to errors of measurement, but is an actual property of the glass. The lower line in Figure 7 was computed by least squares, giving the relative compressibility of the aluminum and the steel. From this and the known absolute compressibility of the aluminum, the cubic compressibility of the steel was

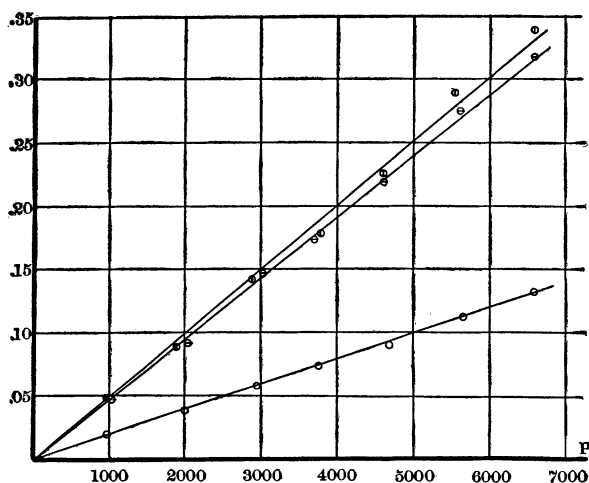


FIGURE 7. Observed relative change of length of steel, and glass or aluminum. The ordinates give the change of length in millimeters, the total length being about 8 cm. ● shows hard Jena glass; — shows soft Jena glass, and ○ the aluminum.

found to be 4.74×10^{-7} , a value somewhat lower than the values found directly for the other specimens of steel. Similarly, the other lines of Figure 7, connecting relative change of length of glass and steel with the pressure, were computed by least squares. The irregularity of the results is too great to warrant the assumption of any other than a linear relation, although the hard Jena glass in particular shows a tendency toward the paradoxical behavior of higher compressibility at higher pressures already remarked in the aluminum. From these constants calculated by least squares, and the compressibility of the

comparison piece of steel determined as above, the compressibility of the glass was found to be:

for Jena glass No. 3880 a 2.17×10^{-6} kgm. per sq. cm.

for Jena glass No. 3883 2.23×10^{-6} kgm. per sq. cm.

The hard glass, contrary to what one might expect, is therefore the more compressible, a result that has already received confirmation by measurements of electrical resistance.

Beside these determinations of the compressibility of glass, it was also necessary to find the compressibility of mercury, in order to find the pressure coefficient of the molecular conductivity of mercury. None of the data at hand reach over a sufficient pressure range for the purpose of this paper, and the data had, therefore, to be extended up to 6800 kgm. The correction introduced by the compressibility of mercury is only 10 per cent of the total change of resistance, so that a highly accurate value of the compressibility was not necessary. The interest of this determination lay rather in finding whether there is any marked decrease of compressibility over the pressure range used. To make this determination, a method was adopted which gives promise of being a considerably better means of determining compressibility even at comparatively low pressures than those methods at present in common use.

The compressibility of mercury at low pressures has been the subject of a number of investigations, and the results which have been obtained recently have been fairly concordant. It is a common feature of all earlier determinations that the mercury has been enclosed in a glass piezometer, the correction for the compressibility of which is 60 per cent of the total effect. The correction for the glass is unusually large in this case because of the comparatively small compressibility of the mercury. For many liquids, the correction for the piezometer is considerably less (6 per cent for water, for example), and the objections urged in the following have proportionally less weight. This correction may be determined in various ways, depending in general on the theory of elasticity, which makes, among others, the assumption of the uniform compressibility of the glass in all directions. Too often, however, the compressibility of the glass has been merely assumed from the work of other investigators on a glass presumably of the same general character as the glass used in the experiments. The correction for compressibility determined by elastic experiments on the same or other pieces of glass seems doubtful in view of the large correction involved. Thus if the behavior of the glass were as

irregular as that observed in the case of the hard Jena glass, discrepancies in the compressibility determined with the same piezometer of as much as 3 per cent are to be expected, at least over any considerable pressure range. Doubtless this uncertain correction for the envelope is the cause of the discordant results previously obtained.

The work of Amagat and de Metz along this line seems the most credible. Each gives the mean of the results with several piezometers, where others have used only one. The results of de Metz ⁴ with four piezometers vary as much as 5 per cent, while those of Amagat ⁵ with seven piezometers vary 2 per cent. The value of Amagat at 20° is 0.00000380 kgm./cm.² while that of de Metz is 0.00000379 kgm./cm.². Lately Richards ⁶ obtained the value 0.00000371, working with a glass piezometer by an electric contact device, but in such a fashion as to eliminate the necessity for calculating the compressibility of the glass, this step being replaced by a calculation from the observed linear compressibility of steel, in which large percentage errors are of much less importance. The values above are for a small pressure range: de Metz and Amagat 50 kgm., and Richards 500 kgm. The results all agree within a unit in the last place, when correction is made for the difference in pressure range.

The only work over a wider range seems to have been done by Carnazzi,⁷ who worked between 0 and 200° and went up to 3000 atmos. He used a glass piezometer, assuming Amagat's mean value for the compressibility of the glass, and a manometer depending in a way not entirely free from objection on the compressibility of water as determined by Amagat. Only two significant figures are given in the results, compressibility at 23° being 0.0000038 from 0 to 500 atmos., and 0.0000034 from 2500 to 3000 atmos. These results must be decreased about 3 per cent, becoming 0.0000037 and 0.0000033 respectively, to reduce from atmospheres to kilograms.

In the present determination, a steel instead of a glass envelope was used. The advantages of a steel over a glass piezometer are manifold. The correction for the compressibility of the steel is only 15 per cent of the total effect against 60 per cent when glass is used. Again, the steel is very much more regular in its elastic behavior than the glass; this is obvious at once from an inspection of the curves showing the compressibility of the glass and of the steel. It has been already stated that the irregular behavior of the glass might introduce

⁴ de Metz, Wied. Ann., **47**, 706 (1892).

⁵ Amagat, C. R., **108**, 228 (1888).

⁶ Richards, loc. cit., p. 51.

⁷ Carnazzi, Nouv. Cim., **5**, 180 (1903).

discrepancies of 3 per cent. Finally, the correction for the glass must be determined from the theory of elasticity, assuming uniform compressibility in all directions. The difficulty of obtaining glass free from internal strain makes the validity of this assumption at least doubtful. Many anomalous results may be explained by this effect. Thus in one case ⁸ an actual increase of the internal volume of the piezometer under hydrostatic pressure has been recorded. On the other hand, the great homogeneity of steel makes its uniform compression *a priori* more probable, and here the probability has been greatly increased by an experimental proof of the uniformity of strain in a piece of rolled steel plate, of the same grade of steel as that used in the mercury piezometer.

The method is essentially a revival of one used by Perkins ⁹ in 1825. Possibly the bad results obtained by Perkins, which were 250 per cent too large, accounts for the subsequent neglect of the method. Several slight modifications were suggested by Professor Sabine, however, so that it has been possible to obtain very satisfactory results. The method consists essentially in observing the extent to which a freely moving piston is pushed into a cylinder containing the liquid to be examined, by the application of hydrostatic pressure all over the exterior of the piston and cylinder. The arrangement used is shown in Figure 8. The containing cylinder C is of

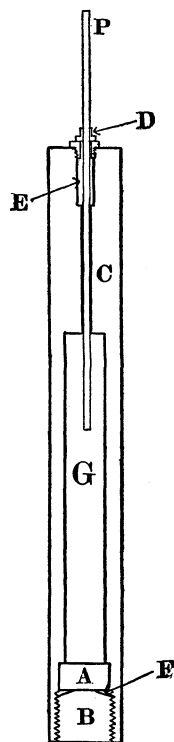


FIGURE 8. Piezometer for determining the compressibility of mercury. C, containing cylinder of steel; G, mercury; P, easily moving piston; D, movable brass ring by which the displacement of the piston is measured. The packing of molasses and glycerine is placed at E. The piezometer is closed at the lower end by the steel plug A, held in place by the screw B. The crack at F is filled with solder.

⁸ M. Schumann, *Wied. Ann.*, **31**, 22 (1887).

⁹ Perkins, *Phil. Trans. Royal Society, London*, p. 324 (1819-1820).
VOL. XLIV. — 18

Bessemer steel $\frac{1}{2}$ in. (1.27 cm.) diameter and $3\frac{1}{2}$ in. (8.89 cm.) long. The piston P is $\frac{1}{8}$ in. (0.16 cm.) in diameter, made in exactly the same way as the piston of the absolute gauge described in a previous paper. The piston accurately fits the hole within 0.0002 or 0.0003 in. (0.00051–0.00076 cm.). The cavity G, which is filled with mercury, is $\frac{1}{4}$ in. (0.635 cm.) in diameter and 2 in. (5.08 cm.) long. The lower end is closed with a plug of steel driven into place and soldered on the outside at F and held additionally by the screw B. The piston P is a slightly looser fit than that used in the absolute gauges, a few ounces without rotation sufficing to displace it. The displacement produced by pressure is indicated by the use of a sliding brass ring at D, exactly as in measuring the change of length of rods. The piezometer was filled by pouring recently distilled mercury through the small hole at the top by a fine glass capillary. The inside of the piezometer was first wet with a few drops of water to insure filling of all the crevices. After filling in this way it was placed under an air pump as an additional precaution against the inclusion of air. The whole was now heated until the mercury rose from the top of the piston hole. The piston, smeared to insure tightness with the same mixture of molasses and glycerine used in the absolute gauge, was inserted and follows the mercury down as it cools. The inside of the enlargement at E was now smeared with molasses, and mercury was poured over the whole to prevent contact of the molasses and the mixture of glycerine and water transmitting the pressure. This packing of viscous molasses very much improved the behavior of the piezometer, reducing the leak past the piston to a minimum. If, however, this packing is used, its protection by the mercury is absolutely necessary, for otherwise the glycerine diffuses through the molasses on each application of pressure, rapidly changing the amount of liquid inside the cylinder.

The method of making the readings was to place the cylinder in the pressure chamber and subject it to hydrostatic pressure all over. By means of the freely moving piston this pressure is transmitted immediately to the interior of the cylinder, the amount of motion of the piston, and so the apparent loss of volume, being indicated by the displacement of the ring D, which is measured after pressure is released and the cylinder removed again from the pressure chamber. This displacement, together with the cross section of the piston and volume of the mercury, gives, therefore, the difference of compressibility between the mercury and the steel of the envelope. The volume of the mercury was obtained by weighing, and the diameter of the piston was measured with a Brown and Sharpe micrometer, the error here not being more than 0.00005 in. on a total of 0.062 in., introducing

a possible error of $\frac{1}{800}$ in the area. The determination of the compressibility of the steel, which must be made independently, takes the place of the determination of the compressibility of the glass in previous work.

A variation of temperature of one degree is equivalent in displacement of the piston to about 50 kgm. The pressure chamber in which the cylinder was placed was inserted in a water bath as nearly as possible at room temperature, and the small variations of this temperature were read to 0.01° after every determination. The temperature at the time of measuring the displacement, which was done with a reading microscope, was also recorded and corrections applied for variations. The observations were carried out at temperatures varying only slightly from 20° , and the final results are for this temperature. The error from temperature variations, which were hardly as much as 0.1° , becomes entirely negligible at the higher pressures, in which the principal interest of this work lay. For accurate work at lower pressures it would, of course, be necessary to take more elaborate temperature precautions.

Another correction necessary to apply is a correction on the measured diameter of the piston, because the piston in advancing into the inner cavity draws with it some of the molasses in the crack between piston and cylinder. The effect of this is to increase the effective diameter of the piston. The question has already been discussed in connection with the absolute gauge and a method given for determining the correction, which, however, is not applicable here. In this case the correction was determined by first smearing the hole in the cylinder with a heavy oil, inserting the piston, and then withdrawing it again. A film of oil adheres to the piston equal approximately to one half of the volume of the oil originally in the crack between piston and cylinder. The quantity of oil thus clinging to the piston was determined by weighing, and the crack in this manner found to be 0.0003 in. (0.00076 cm.) wide. The method of course is very inaccurate, but seemed the only practical way of getting any idea of this small quantity. The total correction thus introduced is only 1 per cent, so that fairly large errors in the correction are unimportant.

It seemed necessary to investigate one other source of possible error before confidence could be placed in the results. There has been expressed a feeling that metals might be porous under high pressures, the experience of Amagat in forcing mercury through 8 cm. of cast steel being adduced as evidence on this point. To test this, a piece of steel from the same piece as the piezometer was weighed before and after subjection to pressure, in an endeavor to detect possible increase of weight from the absorbed liquid. No change of weight of

more than one part in 400,000 could be detected. On a previous occasion a piece of drawn copper had been found to suffer no increase of weight of one part in 1,800,000. It may be confidently expected, therefore, that ordinary commercial bar metal shows no considerable porosity. Amagat's result was probably due to flaws in the casting.

In Figure 9 the observed proportional changes of volume of mercury measured from an arbitrary zero, as in the case of the determination

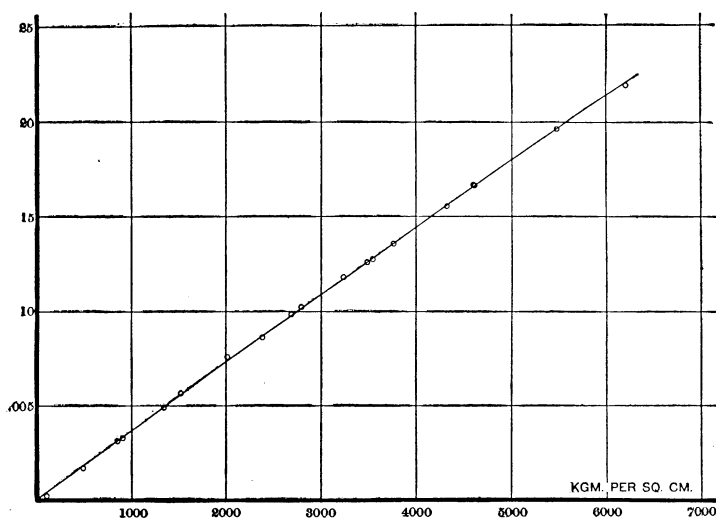


FIGURE 9. The proportional change of volume of mercury, as determined with the piezometer of Figure 8, plotted against pressure.

of the compressibility of rods, are plotted against pressure, measured in the usual way with a mercury resistance. The maximum ordinate corresponds to a displacement of the piston of 1.5 cm. Results obtained with a preliminary piezometer, not so well made as the final one, agree with the curve given within the somewhat larger limits of experimental error. The principal source of error seems to be the inclusion of minute air bubbles. Measurement from an arbitrary zero, determined by backward extrapolation as above, removes this as a consistent source of error, but the measurement of the actual displacement becomes irregular from the lack of certainty with which the piston is returned after release of pressure to the initial position by the comparatively feeble expansive action of the bubble of air. All the precautions described above to remove this bubble appear necessary.

The compressibility of the steel envelope has already been determined, and hence the proportional change of volume of the mercury can be corrected and the true compressibility found. It was assumed that

$$\frac{\Delta V}{V_0} = a + bp + cp^2,$$

and the constants were calculated by least squares. The results are shown in Table V. The constant a has the same significance as in the case of the steel and aluminum rods, the constants b and c alone having significance for the mercury itself. The values found were

$$a = 0.001252,$$

$$b = 3.699 \times 10^{-6},$$

$$c = -1.985 \times 10^{-11}.$$

The compressibility at low pressures is b , 3.70×10^{-6} compared with 3.80×10^{-6} , found by Amagat, de Metz, and Richards, and 3.7×10^{-6} found by Carnazzi. It is to be remarked, however, that the purpose of this investigation was not to find the compressibility at low pressures, only two observations being made at less than 800 kgm. Both the dimensions of the piezometer and the temperature changes make the low pressure values of this determination doubtful. There is, moreover, obvious on inspection of the table a tendency for the low pressure values to lie below the values given by the formula. This would increase the initial compressibility. The experimental error is sufficient, however, to make illusory a more accurate determination of the initial b by passing a curve of the above type through the lower values only. The probable error of a single observation, discarding the first, is $\frac{1}{3}$ per cent at the highest pressure. The probable percentage error of values determined by the formula is 0.25 per cent, discarding the lowest value, or 0.18 per cent, discarding the lowest two.

The departure of the compressibility from constancy is shown by the constant c , which is very small, in fact much smaller than has been found by either Carnazzi or Richards. It may be found from the above formula that the instantaneous compressibility at 2700 kgm. has decreased to 3.58×10^{-6} from its initial value of 3.70×10^{-6} . Carnazzi finds the average compressibility between 2500 and 3000 to be 3.3×10^{-6} against 3.7×10^{-6} between 0 and 500. Richards finds a decrease of compressibility from 3.80×10^{-6} to 3.64×10^{-6} over a pressure range of 500 kgm. However, Richards himself recognized the possibility that his pressure unit might be in error at the higher

TABLE V.
COMPRESSIBILITY OF MERCURY.

Pressure kgm./cm. ²	$\frac{\Delta V}{V_0}$		
	Observed.	Calculated.	Difference.
116	0.000140	0.000168	+28
496	0.000297	0.000308	+11
850	0.000440	0.000439	— 1
916	0.000458	0.000462	+ 4
1346	0.000619	0.000619	0
1536	0.000691	0.000689	— 2
2050	0.000892	0.000875	—17
2380	0.000990	0.000994	+ 4
2690	0.001117	0.001106	—11
2792	0.001157	0.001142	—15
3224	0.001314	0.001297	—17
3492	0.001393	0.001393	0
3550	0.001408	0.001413	+ 5
3760	0.001497	0.001487	—10
4320	0.001679	0.001486	+ 7
4600	0.001796	0.001784	—12
4610	0.001788	0.001787	— 1
5490	0.002097	0.002096	— 1
6216	0.002329	0.002347	+18
$\frac{\Delta V}{V_0} = a + bp + cp^2.$ $a = 0.0012523.$ $b = \log^{-1} 4.5681 - 10.$ $-c = \log^{-1} 9.2977 - 20.$			

pressures. He finds, e. g., for the compressibility of water at 200 and 400 kgm. 42.5 and 39.6 respectively, against 42.4 and 40.6 as found by Amagat. This points, therefore, to an error in Richards' standard in the right direction, and of approximately the right magnitude to bring his result into agreement with the above. It may also be remarked in this connection that the quantity c is essentially a difference of the second order, and that consequently any increase of the pressure range will give a more than proportionate increase in the probable accuracy of c , other things being equal.

The form of steel piezometer described above may be applied with a few obvious modifications to the determination of the compressibility of other liquids than mercury, and even of liquids that attack the steel. In fact, it seems probable that some such form will prove most useful for high pressure work in general, because the forms of glass piezometer in common use become impracticable at high pressures by the cracking of the glass around any pieces of sealed-in platinum, or even by the cracking of the glass alone, when blown into at all complicated shapes.

CONCLUSION.

In this paper there have been presented methods applicable over a wide pressure range for finding the compressibility of solids in the form of rods or tubes, and also of liquids. These methods have been applied to the determination of a few compressibilities which were needed for another purpose. The pressure range employed was 6500 kgm. The compressibilities found were as follows: two pieces of Jena glass

No. 3880 a, 2.17×10^{-6} kgm. per sq. cm.

No. 3883, 2.23×10^{-6} kgm. per sq. cm.

Four pieces of steel: two of Bessemer boiler plate, one of Bessemer rod, and one of tool steel, respectively,

5.298×10^{-7} , 5.303×10^{-7} , 5.16×10^{-7} , and 5.25×10^{-7} .

Another piece of Bessemer by an indirect method, not so accurate, gave 4.7×10^{-7} . Compressibility of commercial aluminum rod, 11.7×10^{-7} . The change of volume of mercury is connected with pressure by the relation

$$\begin{aligned}\frac{\Delta V}{V_0} &= bp + cp^2 \\ b &= \log^{-1} (4.5681 - 10); \\ -c &= \log^{-1} (9.2977 - 20).\end{aligned}$$